

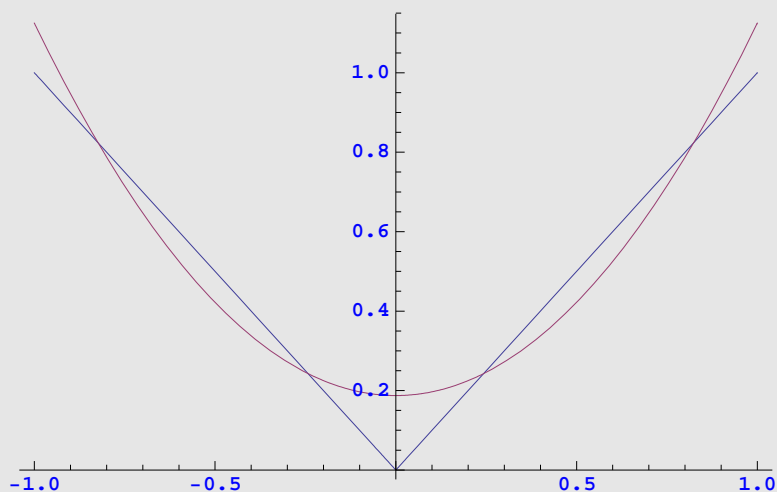
```
IP[f_, g_, w_, L_, R_] := Integrate[f g w, {x, L, R}]
```

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w = 1; L = -1; R = 1; Basis = Table[x^p, {p, 0, 2}]; func = Abs[x];
len = Length[Basis];
OB = Table[0, {i, 1, len}]; ONB = OB;
OB[[1]] = Basis[[1]];
For[n = 2, n <= len, n = n + 1,
  OB[[n]] = Basis[[n]] - Sum[OB[[j]] IP[Basis[[n]], OB[[j]], w, L, R] /
    IP[OB[[j]], OB[[j]], w, L, R], {j, 1, n - 1}];
For[n = 1, n <= len, n = n + 1, ONB[[n]] = OB[[n]] /
  Sqrt[IP[OB[[n]], OB[[n]], w, L, R]]];
ONB
Table[IP[ONB[[i]], ONB[[j]], w, L, R], {i, 1, len}, {j, 1, len}]
coeff = Table[IP[func, ONB[[i]], w, L, R], {i, 1, len}];
phi = Sum[ONB[[i]] * coeff[[i]], {i, 1, len}]
Plot[{func, phi}, {x, -1, 1}]
```

$$\left\{ \frac{1}{\sqrt{2}}, \sqrt{\frac{3}{2}} x, \frac{3}{2} \sqrt{\frac{5}{2}} \left(-\frac{1}{3} + x^2 \right) \right\}$$

$$\{\{1, 0, 0\}, \{0, 1, 0\}, \{0, 0, 1\}\}$$

$$\frac{1}{2} + \frac{15}{16} \left(-\frac{1}{3} + x^2 \right)$$



```

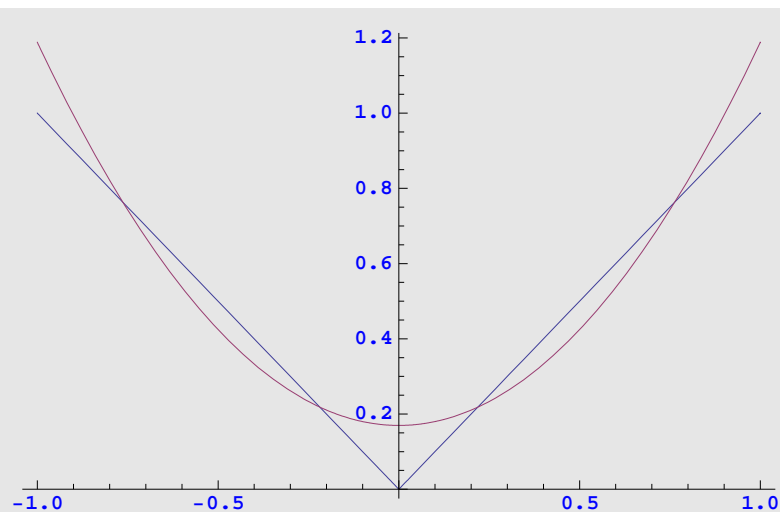
w = (1 - x^2)^(1/2); L = -1; R = 1; Basis = Table[x^p, {p, 0, 2}]; func = Abs[x];
len = Length[Basis];
OB = Table[0, {i, 1, len}]; ONB = OB;
OB[[1]] = Basis[[1]];
For[n = 2, n <= len, n = n + 1,
  OB[[n]] = Basis[[n]] - Sum[OB[[j]] IP[Basis[[n]], OB[[j]], w, L, R] /
    IP[OB[[j]], OB[[j]], w, L, R], {j, 1, n - 1}];]
For[n = 1, n <= len, n = n + 1, ONB[[n]] = OB[[n]] /
  Sqrt[IP[OB[[n]], OB[[n]], w, L, R]]];]
ONB
Table[IP[ONB[[i]], ONB[[j]], w, L, R], {i, 1, len}, {j, 1, len}]
coeff = Table[IP[func, ONB[[i]], w, L, R], {i, 1, len}];
phi = Sum[ONB[[i]] * coeff[[i]], {i, 1, len}]
Plot[{func, phi}, {x, -1, 1}]

```

$$\left\{ \sqrt{\frac{2}{\pi}}, 2 \sqrt{\frac{2}{\pi}} x, 4 \sqrt{\frac{2}{\pi}} \left(-\frac{1}{4} + x^2 \right) \right\}$$

$$\{ \{1, 0, 0\}, \{0, 1, 0\}, \{0, 0, 1\} \}$$

$$\frac{4}{3\pi} + \frac{16 \left(-\frac{1}{4} + x^2 \right)}{5\pi}$$



```

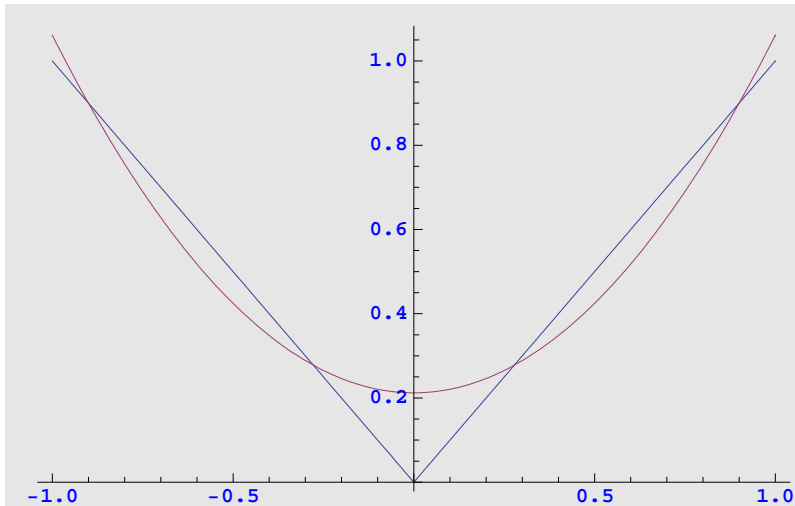
w = (1 - x^2)^(-1/2); L = -1; R = 1; Basis = Table[x^p, {p, 0, 2}]; func = Abs[x];
len = Length[Basis];
OB = Table[0, {i, 1, len}]; ONB = OB;
OB[[1]] = Basis[[1]];
For[n = 2, n <= len, n = n + 1,
  OB[[n]] = Basis[[n]] - Sum[OB[[j]] IP[Basis[[n]], OB[[j]], w, L, R] /
    IP[OB[[j]], OB[[j]], w, L, R], {j, 1, n - 1}];
For[n = 1, n <= len, n = n + 1, ONB[[n]] = OB[[n]] /
  Sqrt[IP[OB[[n]], OB[[n]], w, L, R]]];
ONB
Table[IP[ONB[[i]], ONB[[j]], w, L, R], {i, 1, len}, {j, 1, len}]
coeff = Table[IP[func, ONB[[i]], w, L, R], {i, 1, len}];
phi = Sum[ONB[[i]] * coeff[[i]], {i, 1, len}]
Plot[{func, phi}, {x, -1, 1}]

```

$$\left\{ \frac{1}{\sqrt{\pi}}, \sqrt{\frac{2}{\pi}} x, 2 \sqrt{\frac{2}{\pi}} \left(-\frac{1}{2} + x^2 \right) \right\}$$

$$\{\{1, 0, 0\}, \{0, 1, 0\}, \{0, 0, 1\}\}$$

$$\frac{2}{\pi} + \frac{8 \left(-\frac{1}{2} + x^2 \right)}{3 \pi}$$



```
w = 1; L = -1; R = 1; Basis = Table[xp, {p, 0, 10}]; func = Abs[x];
len = Length[Basis];
OB = Table[0, {i, 1, len}]; ONB = OB;
OB[[1]] = Basis[[1]];
For[n = 2, n <= len, n = n + 1,
  OB[[n]] = Simplify[Basis[[n]] - Sum[OB[[j]] IP[Basis[[n]], OB[[j]], w, L, R] /
    IP[OB[[j]], OB[[j]], w, L, R], {j, 1, n - 1}]];
For[n = 1, n <= len, n = n + 1, ONB[[n]] = OB[[n]] /
  Sqrt[IP[OB[[n]], OB[[n]], w, L, R]]];
ONB
Table[IP[ONB[[i]], ONB[[j]], w, L, R], {i, 1, len}, {j, 1, len}]
coeff = Table[IP[func, ONB[[i]], w, L, R], {i, 1, len}];
phi = Sum[ONB[[i]] * coeff[[i]], {i, 1, len}] // Simplify
Plot[{func, phi}, {x, -1, 1}]
```

$$\left\{ \frac{1}{\sqrt{2}}, \sqrt{\frac{3}{2}} x, \frac{3}{2} \sqrt{\frac{5}{2}} \left(-\frac{1}{3} + x^2 \right), \right.$$

$$\frac{5}{2} \sqrt{\frac{7}{2}} \left(-\frac{3x}{5} + x^3 \right), \frac{105 \left(\frac{3}{35} - \frac{6x^2}{7} + x^4 \right)}{8\sqrt{2}},$$

$$\frac{63}{8} \sqrt{\frac{11}{2}} \left(\frac{5x}{21} - \frac{10x^3}{9} + x^5 \right),$$

$$\frac{231}{16} \sqrt{\frac{13}{2}} \left(-\frac{5}{231} + \frac{5x^2}{11} - \frac{15x^4}{11} + x^6 \right),$$

$$\frac{1}{16} \sqrt{\frac{15}{2}} x \left(-35 + 315x^2 - 693x^4 + 429x^6 \right),$$

$$\frac{6435}{128} \sqrt{\frac{17}{2}} \left(\frac{7}{1287} - \frac{28x^2}{143} + \frac{14x^4}{13} - \frac{28x^6}{15} + x^8 \right),$$

$$\frac{12155}{128} \sqrt{\frac{19}{2}} \left(\frac{63x}{2431} - \frac{84x^3}{221} + \frac{126x^5}{85} - \frac{36x^7}{17} + x^9 \right),$$

$$\frac{46189}{256} \sqrt{\frac{21}{2}}$$

$$\left(-\frac{63}{46189} + \frac{315x^2}{4199} - \frac{210x^4}{323} + \frac{630x^6}{323} - \frac{45x^8}{19} + x^{10} \right) \left. \right\}$$

```
{ {1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0},  
  {0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0},  
  {0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0},  
  {0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0},  
  {0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0},  
  {0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0},  
  {0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0},  
  {0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0},  
  {0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0},  
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0},  
  {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1} }
```

$$\frac{1}{262144} 11 \left(1323 + 85995 x^2 - 286650 x^4 + 584766 x^6 - 566865 x^8 + 205751 x^{10} \right)$$

