Partial Differential Equations – Math 442 C13/C14
Fall 2009
Homework 5 — due October 30

1. **(Strauss 5.2.2.)** Show that $\cos(x) + \cos(\alpha x)$ is periodic if α is a rational number and compute its period. What happens if α is not rational?

2. Define $f(x) = x^3$ on the interval $[0, 1]$. Compute its Fourier sine series and its Fourier cosine series.

3. Consider the function $f(x) = x$ on the interval $[-\pi, \pi]$. Compute the full Fourier series for $f(x)$. Use Parseval's Identity to compute
$$\sum_{n=1}^{\infty} \frac{1}{n^2}.$$

4. Solve the heat equation given by
$$u_t = ku_{xx}, \quad x \in [0, L], t > 0,$$
$$u(x, 0) = x,$$
$$u(0, t) = u(L, t) = 0.$$

5. **(Strauss 5.3.8.)** Let f and g satisfy the same Robin boundary condition at $x = 0$ and the same Robin boundary condition at $x = L$ (i.e., we assume that
$$f'(0) + \alpha f(0) = g'(0) + \alpha g(0) = f'(L) + \beta f(L) = g'(L) + \beta g(L) = 0.)$$
Prove then that
$$(f'(x)g(x) - f(x)g'(x))|_{x=0}^{x=L} = 0.$$
Deduce from this that eigenfunctions of a Robin BVP are orthogonal.

6. Prove that if f has period p, then
$$\int_a^{p+a} f(y)\, dy$$
is independent of a.

7. Consider the infinite list of functions
$$\{1, \cos(x), \cos(2x), \dots, \cos(nx), \dots, \sin(x), \sin(2x), \dots, \sin(nx), \dots\}.$$
Show that this is an orthogonal set of functions on the set $[-\pi, \pi]$, i.e. if we define the inner product
$$\langle f, g\rangle := \int_{-\pi}^{\pi} f(x)g(x)\, dx,$$
then if we choose any two different functions from that list, then their inner product is zero.

8. Let $\{f_n(x)\}$ be any sequence of functions which converge to $f(x)$ uniformly on $[a, b]$. Prove then that $f_n(x)$ converge to f in the L^2 sense as well. Show a counterexample to demonstrate that the converse is false, i.e. that we can have L^2 converge but not uniform. (The term used for this is that uniform convergence is *stronger* than L^2).