Introduction to Differential Equations – Math 286 X1
Fall 2009
Homework 9 — due November 11

1. Determine the fundamental period of the following functions:

   (a) $\cos(2t)$

   (b) $\sin(2\pi t)$

   (c) $\sin^2(t)$

   (d) $\cos(t) + \sin(t)$

2. Is the function $f(t) = \cos(t) + \cos(4t)$ periodic? If yes, demonstrate this by finding a period of the function. Same questions for $g(t) = \cos(t) + \cos(\pi t)$.

3. Let $f(t)$ be a 2π-periodic function defined by

$$f(t) = \begin{cases} 3, & -\pi < t < 0, \\ -4, & 0 < t < \pi, \\ 132, & t = 0, \pi. \end{cases}$$

   Compute its Fourier series.

4. Let $f(t)$ be a 2π-periodic function defined by $f(t) = |t|$ for $t \in [-\pi, \pi]$ and extended periodically elsewhere. Compute its Fourier series.

5. Define f to be the function with period 3 defined as

$$f(t) = t^2, \quad -3/2 < t < 3/2.$$

   Compute its Fourier series.

6. Prove that

$$\int_{-\pi}^{\pi} \cos(nt)\sin(mt) = 0$$

   for any integers n, m. **Hint:** Think about even and odd functions.

7. Prove that

$$\int_{-\pi}^{\pi} \cos(nt)\cos(mt) = \begin{cases} 0, & m \neq n \\ \pi, & m = n. \end{cases}$$

   Hint: For $m \neq n$, use the trig identity

$$\cos A \cos B = \frac{1}{2}(\cos(A + B) + \cos(A - B)).$$

   Why does this calculation fail when $m = n$?