

Introduction to Differential Equations – Math 286 X1
Fall 2009
Homework 7 — due October 21

1. Let

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}.$$

Compute $A + B$. Compute AB and BA . Does $AB = BA$?

2. Let

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}.$$

Compute AB and BA . Does $AB = BA$?

3. Let

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}.$$

Find a matrix B so that $AB = BA$. Now find a second one. Check that they work.

4. For each of the following, determine whether or not the matrix has an inverse, and if it does, compute it:

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \quad \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 1 \\ 2 & 2 \end{pmatrix}, \quad \begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 3 \\ -1 & -3 \end{pmatrix}.$$

5. Solve the system

$$\begin{aligned} x_1' &= -x_1 + 3x_2, \\ x_2' &= -2x_1 + 4x_2, \\ x_1(0) &= 2, \quad x_2(0) = 3. \end{aligned}$$

6. Solve the system

$$\mathbf{x}' = \begin{pmatrix} 1 & 3 \\ 1 & -1 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

7. Solve the system

$$\mathbf{x}' = \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$