Introduction to Differential Equations – Math 286 X1
Fall 2009
Homework 6 — due October 14

1. Consider a mass–spring system where the mass is 4 kg, and the spring constant is 1 kg s^{-1}.

   (a) Denote the position of the mass at time t by $x(t)$. Write down the equation which determines this position.

   (b) Compute the natural frequency of this system.

   (c) Write down the general solution for this system.

   (d) Force the system with the sinusoidal forcing $f(t) = 2\cos(3t)$ N. Write down the general solution for this system. What is the input/output response as defined in class? (Remember that I/O response is defined with an absolute value!!)

   (e) In each of parts (c,d), compute the specific solution we obtain if we assume that the initial position of the system is 0m and initial velocity of the system is 0 m/s.

   (f) Now imagine that we are allowed to force this system with an amplitude of 2N, but we can use any frequency we like, i.e. we choose $f(t) = 2\cos(\omega t)$ N, where ω is chosen by us. Assume that our system will break if the amplitude of the solution ever reaches 100m. Compute the range of ω which will cause this system to break.

2. Take the same system and add friction (right now leave it as an unspecified c).

   (a) Write down the equation which governs the entire system when it is not forced.

   (b) Add a forcing to this system which has the natural frequency (as computed above). What is the input/output response? (Remember that this formula is different when $c > 0$!)

   (c) Finally, fix $c = 1$ kg/s and assume that we force with an unspecified frequency, i.e. $f(t) = F_0\cos(\omega t)$. Find the value of ω which maximizes the input/output response.