Introduction to Differential Equations – Math 286 X1
Fall 2009
Homework 3 — due September 16

1. (a) Solve exactly:

$$\frac{dx}{dt} = 2x - x^2, \quad x(0) = 3.$$

   (b) Now solve exactly:

$$\frac{dx}{dt} = 2x - x^2, \quad x(0) = 1.$$

   (c) Now forget about exact solutions, and use the qualitative analysis we discussed in class to analyze the ODE

$$\frac{dx}{dt} = 2x - x^2.$$

   What is the long-time behavior ($t \to \infty$) of any solution to this equation with $x(0) > 0$? Is this consistent with the answers to #1,2?

2. Consider the autonomous equation $\frac{dy}{dt} = f(y)$. In each case, either demonstrate a function f (drawing the graph of f is fine, no need for a formula), or argue why no such f can exist:

   (a) the equation has no fixed points

   (b) the equation has exactly one fixed point, and it is stable

   (c) the equation has exactly one fixed point, and it is unstable

   (d) the equation has exactly 3 stable and 2 unstable fixed points

   (e) the equation has 5 stable fixed points and 1 unstable fixed point

3. We consider a population of deer and represent the size of the population at time t by $P(t)$. Assume that the rate of growth of the population is proportional to $\sqrt{P}$. We also know that the population at $t = 0$ is 36 deer, and it is increasing at the rate of 12 deer/month. How many deer will there be in one year? In three years? How long will it take to get one million deer?

4. In each of these IVPs, determine whether or not the Existence–Uniqueness Theorem (Theorem 1.3.1 in the book, or in class) guarantees a unique solution:

   (a) $\frac{dy}{dx} = 2x^6 y^2, \quad y(2) = 3,$

   (b) $\frac{dy}{dx} = 2x^6 y^{-2}, \quad y(2) = 3,$

   (c) $\frac{dy}{dx} = 2x^6 y^{-2}, \quad y(1) = 0,$

   (d) $\frac{dy}{dx} = \ln y, \quad y(0) = 0,$

   (e) $\frac{dy}{dx} = \ln y, \quad y(1) = 1.$