Introduction to Differential Equations – Math 286 X1
Fall 2009
Homework 1 Solutions

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1. Verify in each case that the given function is a solution to the differential equation.

   (a) $y' = 5x^4, \quad y(x) = x^5 + 7,$

   (b) $y' = y + 2e^{-x}, \quad y(x) = e^x - e^{-x},$

   (c) $y'' + 4y = 0, \quad y(x) = 7\cos(2x) - 3\sin(2x),$

   (d) $y' + 2xy^2 = 0, \quad y(x) = (1 + x^2)^{-1}.$

   Solution:

   (a) If $y(x) = x^5 + 7$, then $y'(x) = 5x^4$.

   (b) We have that $y'(x) = e^x + e^{-x} = y(x) + 2e^{-x}$.

   (c) We compute that

   $$y' = -14\sin(2x) - 6\cos(2x),$$
   $$y'' = -28\cos(2x) + 12\cos(2x),$$

   and we see that $y'' = -4y$.

   (d) We have that

   $$y'(x) = \frac{-2x}{(1 + x^2)^2} = -2xy^2.$$

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2. In each case, first verify that the given function satisfies the differential equation, then compute the correct value of C so that it also satisfies the initial condition.

   (a) $y' + y = 0, \quad y(x) = Ce^{-x}, \quad y(0) = 3,$

   (b) $y' + 2xy = 0, \quad y(x) = Ce^{-x^2}, \quad y(0) = -1,$

   (c) $y' = x - y, \quad y(x) = Ce^{-x} + x - 1, \quad y(0) = 1.$

   Solution:

   (a) We check that $y' = -Ce^{-x}$ so $y' + y = 0$. Also, plugging in $x = 0$ gives $y(0) = C$, so $C = 3$.

   (b) We check that $y' = -2xCe^{-x^2}$, so $y' + 2xy = 0$. Plugging in $x = 0$ gives $y(0) = C$, so $C = -1$.

   (c) We get $y' = -Ce^{-x} + 1$ and $x - y = x - Ce^{-x} - x + 1$. Plugging in $x = 0$ gives $y(0) = C - 1$, or $C - 1 = 1$, or $C = 2$.

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3. For each of these equations, use your knowledge of derivatives to make an intelligent guess for the solution. Check your guess.

   (a) $y'' = 0,$

   (b) $y' = 3y,$

   (c) $(y')^2 + y^2 = 1.$

   Solution:

   (a) We guess $y(x) = Ax + B$, and this checks.

   (b) We guess $y(x) = Ce^{3x}$, and this checks.

   (c) We see the pattern, and note that $\sin^2 x + \cos^2 x = 1$, so we guess either $\sin x$ or $\cos x$.