

**Introduction to Differential Equations – Math 286 X1**  
**Fall 2009**  
**Homework 10 — due never**

1. Write down the general solution for the heat equation given by

$$u_t = 2u_{xx}, 0 \leq x \leq 4\pi, t > 0,$$
$$u(0, t) = u(4\pi, t) = 0.$$

2. Write down the general solution for the wave equation given by

$$u_{tt} = 4u_{xx}, 0 \leq x \leq 1,$$
$$u(0, t) = u(1, t) = 0.$$

3. Write down the specific solution to the heat equation given by

$$u_t = 2u_{xx}, 0 \leq x \leq 1, t > 0,$$
$$u(0, t) = u(1, t) = 0,$$
$$u(x, 0) = x(1 - x).$$

4. In each of the following cases, you are given a list of functions and a domain. Determine whether or not the list of functions is an orthogonal list. (Recall that we say that a list of functions is an orthogonal list if we can choose any pair of different functions in that list, and they are orthogonal.)

- (a)  $\{1, x, x^2, x^3, \dots, x^n, \dots\}$ , domain  $[0, 1]$ ,  
(b)  $\{\sin(x), \sin(2x), \dots, \sin(nx), \dots\}$ , domain  $[0, \pi]$ ,  
(c)  $\{\sin(x), \sin(2x), \dots, \sin(nx), \dots\}$ , domain  $[0, \pi/2]$ ,  
(d)  $\{\cos(x), \cos(2x), \dots, \cos(nx), \dots\}$ , domain  $[0, \pi]$ .