1 Problems

I. If \( \sin(\theta) = \frac{4}{5} \) what does \( \cos(\theta) \) and \( \tan(\theta) \) equal?

II. Use the law of exponents to rewrite and simplify the expression.

1. \( 8^{5/3} \)

2. \( x(3x^3)^2 \)

3. \( \frac{a^2 \sqrt{b}}{\sqrt{ab}} \)

4. \( \frac{\sqrt[3]{a^2 \sqrt{b}}}{\sqrt[3]{ab^2}} \)

III. Sketch the graph of the function.

1. \( y = 10^{x-2} \)

2. \( y = 2^{-x} \)

3. \( y = 2^{1-x} \)

4. \( y = 4(1 - 2^{x-2}) \)

5. \( y = -3^{x+3} + 5 \)

IV. Find the domain of the function.
1. \( \frac{1 - 2x^2}{1 - e^x} \)

2. \( \frac{1 - 2x^2}{1 - e^{-x^2 + 1}} \)

3. \( \frac{1 + x^2}{2 \sin(x) + 2 \cos(x) + x^2} \)

4. \( \cos(e^{-x^2 + 3}) \)

V. A bacterial culture starts with 1000 bacteria and doubles in size every half hour.
   a) How many bacteria are there after 2 hours?
   b) How many bacteria are there after \( t \) hours?
   c) How many bacteria are there after 40 minutes?

VI A bacterial culture has 1000 bacteria after an hour and 2000 bacteria after 3 hours.
   a) How many bacteria are there after 5 hours?
   b) How many bacteria are there after \( t \) hours?
   c) How many bacteria are there after 30 minutes?
2 Solutions

I. If \( \sin(\theta) = \frac{4}{5} \) what does \( \cos(\theta) \) and \( \tan(\theta) \) equal?

Answer:
\[
\cos(\theta) = \frac{3}{5} \\
\tan(\theta) = \frac{4}{3}
\]

II. Use the law of exponents to rewrite and simplify the expression.

1. \( \frac{8^{5/3}}{\sqrt[3]{ab}} \)
   Answer:
   \[
   \frac{8^{5/3}}{\sqrt[3]{ab}} = \frac{a^{2b^{1/2}}}{(ab)^{1/4}} = \frac{a^{2b^{1/2}}}{a^{1/4}b^{1/4}} = a^{2-1/4}b^{1/2-1/4} = a^{7/4}b^{1/4} = \sqrt[4]{a^7b}
   \]

2. \( x(3x^3)^2 \)
   Answer:
   \[
   x(3x^3)^2 = x(9x^6) = 9x^7
   \]

3. \( \frac{a^2\sqrt{b}}{\sqrt[3]{ab}} \)
   Answer:
   \[
   \frac{a^2\sqrt{b}}{\sqrt[3]{ab}} = \frac{(ab)^{1/2}}{a^{1/3}b^{1/3}} = a^{1/2-1/3}b^{1/2-1/3} = a^{1/6}b^{-1/2} = \frac{\sqrt[6]{a}}{\sqrt{b}}
   \]

III. Sketch the graph of the function.

1. \( y = 10^{x-2} \)
2. \( y = 2^{-x} \)

3. \( y = 2^{1-x} \)
4. \( y = 4(1 - 2^{x-2}) \)

5. \( y = -3^{x+3} + 5 \)
IV. Find the domain of the function.

1. \( \frac{1 - 2x^2}{1 - e^x} \)
   Answer:

   \[
   1 - e^x \neq 0 \\
   1 \neq e^x \\
   0 \neq x
   \]

   Domain: \( x \neq 0 \)

2. \( \frac{1 - 2x^2}{1 - e^{x^2 - 1}} \)
   Answer:

   \[
   1 - e^{x^2 - 1} \neq 0 \\
   1 \neq e^{x^2 - 1} \\
   0 \neq x^2 - 1 \\
   x \neq \pm 1
   \]

   Domain: \( x \neq \pm 1 \)
3. \(\frac{1+x^2}{2\sin(x)+2\cos(x)+x^2}\)
   Answer:
   \(2\sin(x)+2\cos(x)+x^2 \neq 0\)
   \(2^x\) is never equals zero, therefore the function exists for all values of \(x\)
   Domain: \(\mathbb{R}\)

4. \(\cos(e^{-x^2+3})\)
   Answer:
   \(\cos(x), e^x, -x^2 + 3\), are defined for all \(x\).
   Domain: all real values of \(x\)

V. A bacterial culture starts with 1000 bacteria and doubles in size every half hour.
a) How many bacteria are there after 2 hours?

Answer:
It will double four times in 2 hours, and be \(2^4 = 16\) times as large.
There will be 16000 bacteria.

b) How many bacteria are there after \(t\) hours?

Answer:
\(B(t) = 10002^t\)

c) How many bacteria are there after 40 minutes?

Answer:
40 mins = \(2/3\) hours
\(B(2/3) = 10002^{4/3} = 2000\sqrt[3]{2}\)

VI A bacterial culture has 1000 bacteria after an hour and 2000 bacteria after 3 hours.
a) How many bacteria are there after 5 hours?

Answer:
From the data we can see it doubles every 2 hours, therefore there will be 4000 bacteria after 5 hours.
b) How many bacteria are there after \( t \) hours?

Answer:
\[ B(1) = 1000 \text{ and } B(3) = 2000 \]

\[
egin{align*}
2000 &= C \cdot a^3 \\
1000 &= C \cdot a^1 \\
\frac{2}{a^2} &= a = \sqrt{2}
\end{align*}
\]

\[ 1000 = C \cdot a^1 \implies 1000 = C \cdot \sqrt{2} \implies C = \frac{1000}{\sqrt{2}} \]

\[ B(t) = \frac{1000}{\sqrt{2}} \cdot \sqrt{2} = \frac{1000}{\sqrt{2}} \cdot 2^{t/2} = 1000 \cdot 2^{(t-1)/2} \]

c) How many bacteria are there after 30 minutes?

Answer:
30 minutes = 1/2 hours
\[ B(0.5) = 1000 \cdot 2^{(0.5-1)/2} = 1000 \cdot 2^{-1/2} = \frac{1000}{\sqrt{2}} = 500\sqrt{2} \]