Day 40 Worksheet

Periodic function, \(-l < x < l \Rightarrow\) use Fourier series, \(c_n = \text{amplitude at } k\text{-th frequency}\)

Non-periodic function, \(-\infty < x < \infty \Rightarrow\) use Fourier transform, \(F(k) = \text{amplitude at freq. } k\)

1. Verify the transforms (5), (6) on p. 326.

2. Note: \(\mathcal{O}\) transform (10) is just (5) in reverse (put \(F(k) = 2\pi S(k)\) into (3))
   \(\mathcal{O}\) transform (11) is done on p. 326

3. Verify properties (i) - (vi) on p. 327. Hint for (ii): substitute \(\frac{dF}{dk}\) into (3) in place of "\(F\)"

Property (i) says the Fourier transform takes differentiation to multiplication
   "(i)
   "(ii)
   "(iii)
   "(iv)
   "(v)

And Property (v) says the Fourier transform is a linear operation.
Space for calculations:
Fix $t > 0$, $\tilde{k} > 0$.

Let $f(x) = S(x, t)$, the source fn. for diffusion eq. with diffusivity $\tilde{k}$.

$$f(x) = \frac{1}{\sqrt{4\pi \tilde{k}t}} e^{-x^2/4\tilde{k}t}.$$ 

Find the Fourier transform. \underline{HINT:} $f(x) = g\left(\frac{1}{\sqrt{\tilde{k}t}} x\right) / \sqrt{4\pi \tilde{k}t}$.

where $g(x) = e^{-x^2/2}$. 