This course will bring together hyperbolic and euclidean geometry, ergodic theory, and number theory. Our main focus will be on the group $SL(2, \mathbb{R})$, and its action on the homogenous space $SL(2, \mathbb{R})/SL(2, \mathbb{Z})$ by left multiplication.

$SL(2, \mathbb{R})/SL(2, \mathbb{Z})$ is an object of central interest in many areas of mathematics, it is known variously as the modular surface; the universal elliptic curve; the space of lattices; and the space of tori.

We will do a gentle overview of two dimensional hyperbolic geometry and the basics of ergodic theory, then proceed to more advanced topics. One of our main goals will be to prove the Howe-Moore theorem on decay of matrix coefficients, and understand the applications to geodesic and horocycle flows, lattice point counting problems, and diophantine approximation. If time permits, we will discuss basic examples of Ratner’s theorem.

No previous familiarity with hyperbolic geometry, ergodic theory or number theory will be assumed. The only prerequisite is a basic knowledge of measure theory and functional analysis and undergraduate group theory and linear algebra.