

MATH 595
FINITE MODEL THEORY, MEASURE THEORY, AND
STRUCTURE OF POLISH GROUPS

Instructor: Sławomir Solecki

Time: MWF 2–2:50, Fall 2009

The course will focus on connections between the structure and dynamics of particular Polish groups and finite model theory, concentration of measure, and combinatorics.

In the first half of the course, we will present the fundamental theorem of Herwig and Lascar in finite model theory/combinatorics on extending partial isomorphisms [1]. We will apply this theorem to obtain results on the structure and dynamics of the group of all isometries of the Urysohn metric space and some groups of automorphisms of countable structures. In the second part of the course, we will study various dynamical properties of Polish groups in the style of [4]: extreme amenability, existence of non-trivial unitary representations, Lévy property, etc. Even though we will be mainly concerned with examples, we will require interesting mathematical theory, which we will develop as needed: concentration of measure [2] and combinatorial applications of algebraic topology [3].

Background: basic metric topology, basic analysis.

Literature:

1. B. Herwig, D. Lascar, *Extending partial automorphisms and the profinite topology on free groups*, Trans. Amer. Math. Soc. 352 (1999), 1985–2021.
2. M. Ledoux, *The Concentration of Measure Phenomenon*, Amer. Math. Soc., 2001.
3. J. Matoušek, *Using the Borsuk–Ulam Theorem*, Springer, 2003.
4. V. Pestov, *Dynamics of Infinite-dimensional Groups: the Ramsey–Dvoretzky–Milman Phenomenon*, Amer. Math. Soc., 2006.