In recent years, there have been a number of key developments at the intersection of combinatorics, algebraic geometry and representation theory that concern the Grassmannian (and its generalizations). These include:

1. Total positivity
2. Cluster algebras
3. Schubert calculus (real and complex)
4. Kazhdan-Lusztig theory
5. Gröbner and toric degenerations
6. Tropicalization
7. The Wronskian map

I will introduce and discuss the Grassmannian case of these topics, where the known combinatorics is richest, and where much research is left to be done. No prior background is assumed.