MATH 595 : AN INTRODUCTION TO MODULAR FORMS

Time/Location: 11-11:50 MWF, Altgeld Hall room 441.
Pre-requisites: Complex analysis, mathematical maturity, and some exposure to number theory.
Textbook: A variety of sources will be used, but Diamond and Shurman’s “A First Course in Modular Forms” will be used most frequently.
Description: Modular forms are holomorphic functions $f$ that transform like
\[ f \left( \frac{az + b}{cz + d} \right) = (cz + d)^k f(z) \]
for lots of linear fractional transformation $\frac{ax + b}{cx + d}$ with $ad - bc = 1$. These functions frequently have Fourier expansions
\[ f(z) = \sum_{n=0}^{\infty} a(n)q^n \]
and the Fourier coefficients capture information about partitions, elliptic curves, values of $L$-functions, quadratic forms, and class numbers. Modular forms live in finite-dimensional vector space, and so proving identities between them involves only a finite computation. A number of surprising results follow from this theory. Here are some samples:

- If $\sigma_k(n) = \sum_{d|n} d^k$, then
  \[ \sigma_7(n) = \sigma_3(n) + 120 \sum_{k=1}^{n-1} \sigma_3(k)\sigma_3(n-k). \]

- Let
  \[ \sum_{n=1}^{\infty} \tau(n)q^n = q\prod_{n=1}^{\infty}(1-q^n)^{24}. \]
  If $\gcd(m, n) = 1$, then $\tau(mn) = \tau(m)\tau(n)$.

- Let
  \[ \sum_{n=1}^{\infty} a(n)q^n = q\prod_{n=1}^{\infty}(1-q^n)^2(1-q^{11n})^2. \]
  If $p \geq 5$ is an prime, then $a(p) = -\sum_{n=0}^{p-1} \left( \frac{n^3 - 432n + 8208}{p} \right)$.

This course will cover the basic theory of modular forms for congruence subgroups, Hecke operators, and further topics chosen based on the available time and student interest.