In the first part of the course, we will introduce the time-frequency analysis by presenting Lacey-Thiele’s proof on the boundedness of the bilinear Hilbert transforms. Many concepts and technicalities in classical harmonic analysis will be reviewed in this part. The second part of the course will focus on the interface between analytic number theory and harmonic analysis, which will be the main theme of our course. The following topics are planned to be covered.

- Introduction to the time-frequency analysis
- Proof of $L^p$ boundedness of the bilinear Hilbert transforms via the time-frequency analysis
- The exponential sums
- Hardy-Littlewood circle method
- Counting the lattice points on spheres (an application of Hardy-Littlewood circle method)
- $L^p$ boundedness of the discrete Hilbert transforms along polynomial curves
- $L^p$ boundedness of the bilinear Hilbert transforms along polynomial curves (if time permits)

Lectures: TR 11:00am to 12:20 in 141 Altgeld.

Textbook: There is NO textbook for the course. Some references will be given in class.

Exams: No exams.

Prerequisites: Solid knowledge of real analysis and complex analysis.

Suggested Background: Some basic knowledge of harmonic analysis (Math 545) and elementary number theory (any undergraduate course on elementary number theory). It is possible to take this course for students who have no suggested background.