The course will be a gentle introduction to $L^2$-cohomology and $L^2$-invariants for groups and spaces, and a discussion of why they are important. These are the “Hilbert space versions” of the usual homology and of the usual Betti numbers. The subject is very modern and is related to geometry, topology of manifolds and other spaces, group theory, group algebras, analysis, measure theory. Some familiarity with homological algebra and algebraic topology might be helpful, but there are no required prerequisites, we will try to make it gentle.

No textbook will be required for the course. This course will be a mixture of various subjects. One reference might be the book and articles by Lück, and the article by Cheeger and Gromov, on $L^2$-cohomology. We will also discuss an important work by Gaboriau which generalizes $L^2$-invariants from groups to equivalence relations. The purpose of this course is to present a reasonable overview of the subject. Other possible references are two books by Dixmier: “C*-algebras” and “Von Neumann Algebras”.

Important, and interesting, open problems in these areas will be discussed. One is the Atiyah Conjecture, which can be thought of as an analytic analog of the Kaplansky’s Zero-Divisors Conjecture coming from ring theory. The Hanna Neumann Conjecture, a question about free groups, also has restatements in this language. This allows asking more general questions: submultiplicativity of $\ell^2$-numbers. Still another is the Singer Conjecture, related to the topology of manifolds.