MORSE THEORY
with applications to Riemannian geometry and symplectic topology
Fall 2011

Instructor: Ely Kerman

Times: Tuesday-Thursday 11:00 - 12:20

Location: 1 Illini Hall

Overview: Morse theory is the study of the profound relationship between the topology of a space and the behavior of the functions defined on it. It is an extremely powerful tool which plays an important role in many areas of geometry and topology. In this course we will first discuss the basic machinery of Morse theory starting with the material described in Milnor’s classic text. We will then discuss the modern formulation of these ideas due to Thom, Smale, Witten and Floer. This goes under the name of Morse homology, and is the (finite-dimensional) model of Floer homology.

The remainder of the class will be devoted to applications of these tools. The first set of applications will involve the theory of geodesics on Riemannian manifolds (Morse’s original motivation). This will culminate in a discussion of (Bott’s proof of) the Bott periodicity Theorem. The second set of applications involve questions from symplectic topology. We will use stable Morse theory to prove Arnold’s Conjecture concerning the fixed points of Hamiltonian diffeomorphisms on the torus. Generalizations of this conjecture continue to motivate a considerable amount of activity in the field and many of them remain open. If time permits, we will also use Morse theory to prove the (recently settled) Conley Conjecture.

Prerequisites: The prerequisites for this class are basic differential topology and algebraic topology at the level of Guillemin and Pollack’s book Differential Topology and Vassiliev’s book Introduction to Topology. If you have taken Math 518 you should be well equipped for this class.

References (Suggested):