

The absolute arithmetic continuum and the unification of all numbers great and small  
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In his monograph *On Numbers and Games* [1], J. H. Conway introduced a real-closed field containing the reals and the ordinals as well as a great many less familiar numbers including  $\omega$ ,  $\omega/2$ ,  $1/\omega$ ,  $\sqrt{\omega}$  and  $\omega-\pi$  to name only a few. Indeed, this particular real-closed field, which Conway calls *No*, is so remarkably inclusive that, subject to the proviso that numbers--construed here as members of ordered "number" fields--be individually definable in terms of sets of NBG, it may be said to contain "All Numbers Great and Small." In this respect, *No* bears much the same relation to ordered fields that the system of real numbers bears to Archimedean ordered fields.

In a number of earlier works (cf. [2], [3], [4] and [5]), we suggested that whereas the real number system should be regarded as constituting an arithmetic continuum modulo the Archimedean axiom, the system of surreal numbers may be regarded as a sort of absolute arithmetic continuum modulo NBG. In this paper, we will outline some of the properties of the system of surreal numbers we believe lend credence to this mathematico-philosophical thesis, and draw attention to important respects in which the theory of surreal numbers may be regarded as a vast generalization of portions of Cantor's theory of ordinals, a generalization that also provides a setting for Abraham Robinson's infinitesimal approach to analysis as well as for the non-Cantorian theories of the infinite (and infinitesimal) pioneered by Giuseppe Veronese, Tullio Levi-Civita, David Hilbert and Hans Hahn in connection with their work on non-Archimedean ordered algebraic and geometric systems and by Paul du Bois-Reymond, Otto Stolz, Felix Hausdorff and G. H. Hardy in connection with their work on the rate of growth of real functions, theories that have been enjoying a resurgence of interest by logicians in recent decades.

[1] J. H. Conway, *On numbers and games*, Academic Press, 1976.

[2] P. Ehrlich, *Universally extending arithmetic continua*, in *Le Continu Mathematique, Colloque de Cerisy* (H. Sinaceur and J.M. Salanskis, editors), Springer-Verlag France, Paris, 1992, pp. 168-178.

[3] \_\_\_\_\_, *Continuity*, in *Encyclopedia of philosophy, second edition* (D. M. Borchart, Editor in Chief), Macmillan Reference USA, 2005, Volume 2, pp. 489-518.

[4] \_\_\_\_\_, *The Absolute Arithmetic Continuum, Synthese* (forthcoming).

[5] \_\_\_\_\_, *The absolute arithmetic continuum and its Peircean counterpart*, in *New Essays on Peirce's Mathematical Philosophy* (M. Moore, editor), Open Court Press (forthcoming).