GAPS FOR SADDLE CONNECTIONS, OR, THE L-SHAPED OCTAGON

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We are studying the fine distribution properties of saddle connections on the octagon. We consider a regular octagon with opposite sides identified by translations (see Figure 1) and define a saddle connection to be a straight line path that connects corners. These trajectories are of great interest in the study of billiard paths in the octagon and associated right triangles. Motivated by [1], we are interested in doing both numerical experiments and theoretical work to understand the distribution of the gaps in slopes (equivalently, angles) between saddle connections of length $R$ (as $R \to \infty$).

To conduct numerical experiments, it is crucial to have an efficient algorithm to generate the holonomy vectors of the saddle connections. The holonomy vector of a saddle connection is simply the vector whose horizontal and vertical components are the horizontal and vertical distances traveled by the saddle connection respectively. S. Lelievre constructed an effective algorithm to generate the saddle connections of an L-shaped polygon, and we are attempting to adapt his program to our purposes. For this, the first step is to find an appropriate affine map to transform the octagon into a L-shaped table. After some study, we have found that this affine map can be constructed by applying the matrix

$$
\begin{pmatrix}
1 & -1 - \sqrt{2} \\
0 & 1
\end{pmatrix},
$$

and this yields the L-shaped table illustrated below in Figure 2.

Currently, we are working to complete our program to generate saddle connections and study the angles in between them. After we obtain some numerical results, we will attempt to conjecture and prove a theorem about the limiting distribution.

REFERENCES

Figure 1. The Octagon. Opposite sides are identified, and the singular point is at the corner. Note that all eight corners collapse to the same point under identification, and as in the $L$, the total angle is $6\pi = 8 \times \frac{3\pi}{4}$. The dashed lines are a selection of saddle connections.

Figure 2. The Octagonal $L$. The horizontal long side of the $L$ has length $\sqrt{2}$, and the vertical long side has length $1 + \sqrt{2}$. The bottom square is a $1 \times 1$ square. The singular point is marked with a dot, and the total interior angle is $6\pi$. Identifications of sides are as marked by color. The dashed lines are a selection of saddle connections. The purple dashed line is a saddle connection that starts at $(0,0)$, goes to $(\sqrt{2}, 1/2)$ then continues from $(0, 1/2)$ to $(\sqrt{2}, 1)$.