This summer I worked on two separate problems during REGS. Each are described below.

1 Tree Packing

This is the problem I spent most of my time on this summer. It was originated by A. Garca, C. Hernando, F. Hurtado, M. Noy, J. Tejel (1997, 2002) and was presented this summer by Hemanshu Kaul.

**Definition** We say that two graphs $G_1, G_2$ pack into a graph $G$ if there is a mapping $f_1$ of the vertices of $G_1$ to the vertices of $G$ and a mapping $f_2$ of the vertices of $G_1$ to the vertices of $G$ such that if $uv$ is an edge in $G_1$ then $f_1(u)f_1(v)$ is an edge in $G$ (similarly for edges of $G_2$). Also, for a packing if $f_1(u_1) = f_2(u_2)$ and $f_1(v_1) = f_2(v_2)$ then only one of $u_1v_1$ and $u_2v_2$ can be edges. In other words, in $G$ the two graphs share no edges.

**Background:** Hedetniemi, Hedetniemi, and Slater [HHS] proved that any two non-star trees with $n$ vertices pack into $K_n$.

**Definition** A graph is planar if there is a way to draw it on the plan such that no two edges intersect except at a vertex.

**Conjecture:** (Garcia et al. [GHHNT1,GHHNT2]) For any non-star trees $T_1$ and $T_2$ on $n$ vertices, there is a planar $n$-vertex graph $G$ such that $T_1$ and $T_2$ pack into $G$.

Two main results were proved this summer

**Result:** If $T_1$ is a double star and $T_2$ is a non-star tree, then there is a planar $n$-vertex graph $G$ such that $T_1$ and $T_2$ pack into $G$.

**Result:** If both $T_1, T_2$ are non-star caterpillars, then there is a planar $n$-vertex graph $G$ such that $T_1$ and $T_2$ pack into $G$.

I am currently working to extend the previous result to: if $T_1$ is a non-star caterpillar and $T_2$ is a non-star tree, then there is a planar $n$-vertex graph $G$ such that $T_1$ and $T_2$ pack into $G$. 
2 Defining Sets

This problem was originated by E.S. Mahomoodian and R. Naserasr

**Definitions:** For a given graph $G$ a defining set is a set of vertices $V' \subseteq V(G)$ and a proper coloring $\pi'$ of $V'$ such that there exists a unique extension $\pi$ of $\pi'$ to $V(G)$ that is a proper and optimal coloring of $G$. A critical set is a minimal defining set. We let $d(G, \pi)$ be the size of the smallest critical set of $G$ whose unique extension is $\pi$ where $\pi$ is an optimal, proper coloring. Similarly, $D(G, \pi)$ is the size of the largest critical set of $G$ whose unique extension is $\pi$. $\overline{d}(G) = \min d(G, \pi)$ over all proper colorings $\pi$, $\overline{D}(G) = \min D(G, \pi)$ over all proper colorings $\pi$.

**Background:** This problem is often researched by those interested in designs and latin squares. Defining sets have been looked at for uniquely colorable graphs, Hajiabolhassan et al [HMTZ] showed that a graph is critically $\chi(G) - 1$-uniform if and only if $G$ is uniquely colorable.

**Comments:** in Mahmoodian and Naserasr [MN] the following results are proved (1) For any graph $d(G) \geq |V| - |E|/(\chi(G) - 1)$ (2) $d(C_{2n+1}) = n + 1$ (3) If $\chi(G) \leq n$ then $d(GXK_n) \geq |V(G)|(n-1) - 2|E(G)|$.

**Comments:** In Cooper and Kirkpatrick [CK] the following results are proved (1) $\overline{d}(C_{2n+1}) = 2n - 1$

(2) $\overline{D}(C_{2n+1}) = n + 2$ if $2n + 1 \equiv 1 \mod 4$ and $n + 1$ otherwise

(3) $\overline{D}(C_{2n+1}) = 2n$

This was the topic I presented in REGS. The following were the four problems worked on related to this area. **Problem 1:** Characterize all critically $k$-uniform graphs **Problem 2:** How does allowing more colors effect $\overline{d}(G)$ **Problem 3:** Can you find general bounds in terms of other graph parameters **Problem 4:** Can you say anything about other families of graphs

3 Conclusion

I hope to continue research on both of these topics. The author acknowledge support from National Science Foundation grant DMS 08 − 38434 EMSW21−MCTP: Research Experience for Graduate Students.

4 References


[FGK1] F. Frati, M. Geyer, and M. Kaufmann, Packing and squeezing subgraphs into


