FALL 2016

MATH 500

Abstract Algebra

Section C1, CRN 30815

9-9:50 AM MWF

Professor Fernandes

Course Description: This course covers the basic facts about groups, rings, modules and Galois Theory.

1. Group Theory. [Approximately 4.5 weeks]
   (a) Isomorphism theorems for groups.
   (b) Group actions on sets; orbits, stabilizers. Application to conjugacy classes, centralizers, normalizers.
   (c) The class equation with application to finite p-groups and the simplicity of A5.
   (d) Composition series in a group. Refinement Theorem and Jordan-Hölder Theorem. Solvable and nilpotent groups.
   (e) Sylow Theorems and applications.

2. Commutative rings and Modules. [Approximately 5 weeks]
   (a) Review of subrings, ideals and quotient rings. Integral domains and fields. Polynomial rings over a commutative ring.
   (b) Euclidean rings, PID's, UFD's.
   (c) Brief introduction to modules (over commutative rings), submodules, quotient modules.
   (d) Free modules, invariance of rank. Torsion modules, torsion free modules. Primary decomposition theorem for torsion modules over PID's.
   (e) Structure theorem for finitely generated modules over a PID. Application to finitely generated Abelian groups and to canonical form of matrices.
   (f) Zorn's lemma and Axiom of Choice (no proofs). Application to maximal ideals, bases of vector spaces.

3. Field Theory. [Approximately 5 weeks]
   (a) Prime fields, characteristic of a field.
   (b) Algebraic and transcendental extensions, degree of an extension. Irreducible polynomial of an algebraic element.
   (c) Normal extensions and splitting fields. Galois group of an extension.
   (d) Algebraic closure, existence and uniqueness via Zorn's Lemma. Finite fields.
   (e) Fundamental theorem of Galois theory.
   (f) Examples of Galois extensions. Cyclotomic extensions.
   (g) If time permits, application of Galois theory to solution of polynomial equations, symmetric functions and ruler and compass constructions.

2) Lectures Notes to be furnished during the semester.
FALL 2016
MATH 502
INTRODUCTION TO COMMUTATIVE ALGEBRA

Professor S. P. Dutta
11:00 – 12:20 Tu-Th

This course is intended mainly for students who are going to specialize in Commutative Algebra, Algebraic Geometry, Algebraic K-theory and Algebraic Number Theory.

In this course we will mainly focus on Noetherian rings and modules. The topics will include: Primary decomposition, Artin-Rees Lemma, Flatness, Completion, Hilbert-Samuel Polynomial, Dimension Theory, Integral extensions, Going-up and Going-down theorems, Noether’s Normalization (its geometric interpretation), Regular rings and the notion of depth. We would also like to study Cohen-Macaulayness if time permits.

Prerequisite: Math500, 501

Recommended text: Commutative ring theory by H. Matsumura
MATH 506: REPRESENTATION THEORY, FALL 2016

Course Meets: 9:30-10:50 Tu Th,

Instructor: Thomas Nevins (nevins@illinois.edu)

Prerequisites: Math 500.

Course Web Page: http://www.math.uiuc.edu/~nevins/courses/aut16/m506.html

Representation theory is part of the algebraic study of symmetry. What sets it
apart is its focus on the specific study of representations: usually, vector spaces on
which symmetries act. Representations are ubiquitous in the study of symmetry,
since symmetries of physical systems or geometric objects yield representations on
spaces of states or fields (in physics) or functions (in geometry and analysis).

The course will constitute an introduction to representation theory.

Much of the course will be occupied with the study of representations of finite
groups (in the "non-modular" situation, i.e. where the vector spaces live over fields
of nonzero or at least sufficiently large characteristic to simplify life). The represen-
tation theory of finite groups is easy enough to avoid most technical complications
and have good answers to many fundamental questions, but subtle enough that
it will allow us to illustrate many basic phenomena that pervade representation
theory of all kinds.

Later in the course, we will choose, depending on the interests of the class and
as time permits, a more advanced topic that allows us to appreciate some of the
complexity and difficulty of representation theory. Possibilities might include the
representation theory of compact Lie groups or noncompact real groups; representa-
tions of finite groups of Lie type, induction and restriction, and the Harish-Chandra
philosophy; the Deligne-Lusztig construction of characters of finite groups of Lie
type; or the representation theory of complex semi simple Lie algebras and its
relation to the geometry of flag varieties.

Students should plan to do some reading outside of class. While grading of the
course will be relaxed in attitude, there will be regular assigned homework to offer
students an opportunity to test their mastery. [Students who wish to have more
information before deciding whether to register are invited to contact Professor
Nevins.]
FALL 2016  
MATH 512  
ALGEBRAIC GEOMETRY I

Time: MWF 11:00–11:50  
Room: TBA  
Instructor: Sheldon Katz  
Text: Algebraic Geometry, R. Hartshorne, Graduate Texts in Mathematics 52, Springer NY 1977  
Prerequisites: Abstract Algebra II (Math 501), or Introduction to Algebraic Geometry (Math 511), or permission of the instructor

This course introduces Algebraic Geometry from both the algebraic and geometric viewpoints. Affine and projective varieties will be developed from both the classical viewpoint as well as using the language of schemes and sheaves.

This is the first semester of a one-year sequence in Algebraic Geometry, with Algebraic Geometry II planned to run in Spring 2017.

The course will focus on the first two chapters of the text: Varieties, and Schemes, primarily the second chapter on Schemes. The text will be frequently supplemented with additional materials designed to enhance geometric intuition.

Comments on prerequisites: students can profitably take this course after either Abstract Algebra II (Math 501) or Introduction to Algebraic Geometry (Math 511). It is not necessary to have taken both courses. We will be freely using the language of commutative algebra for the most part, including notions such as prime ideals, localization, and tensor product. This is not the only route to algebraic geometry but is a choice being made largely by the choice of textbook. Many of the ideas from Math 511 are developed in a more general and abstract setting in Math 512, so 511 will be helpful. But since the ideas will be developed independently, 511 is not necessary. Familiarity with Riemann Surfaces and Algebraic Curves (Math 510) is not necessary but will be helpful, as a smooth 1-dimensional projective variety over the complex numbers is equivalent to a compact Riemann surface.
Math 518, Differentiable Manifolds I (Fall 2016)

Instructor: Pierre Albin
Office: Illini Hall 237
Email: palbin [at] illinois .edu

Lectures: TBA
Office Hours: TBA


Recommended Texts:
- Lee, Introduction to Smooth Manifolds.
- Bredon, Topology and Geometry.
- Guillemin, Pollack, Differentiable Topology.
- Gallot, Hulin, Lafontaine, Riemannian Geometry.

Assignments: There will be homework each week. You are allowed (and encouraged) to work with other students while trying to understand the homework problems. However, the homework that you hand in should be your work alone. Late homework will not be accepted, but the lowest score will be dropped.

Holidays: Classes begin on August 22 and end on December 7. There will be no classes on:
- Labor Day, September 5
- Thanksgiving break, November 21 - November 25

Grading percentages:
- Problem sets (35%)
- Midterm exam (25%)
- Final exam (40%)

Description:
This is a first graduate course on smooth manifolds. Smooth manifolds arose from the maelstrom of non-Euclidean geometries in the nineteenth century and now form the basic language of many topics in, e.g., physics, group theory, number theory, as well as being the central topic in topology and geometry. In this course we will start from the definition and build up some of the most important constructions associated with it, such as vector bundles, tangent and cotangent vectors, submanifolds, etc. We will also discuss the extension of multivariable calculus from Euclidean spaces to manifolds.
Course Description: Lie groups and Lie algebras were discovered by Sophus Lie in the second half of the 19th Century. His motivation was to design a Galois Theory for differential equations, which would allow to determined if one could solve a differential equation, as well as to find systematic methods of integration. Soon after, it was realized that Lie groups and algebras play a fundamental role in many other areas of mathematics. Nowadays, they are recognized as a basic tool in mathematics and its applications. This course will provide a solid introduction to the basic of Lie groups and Lie algebras, as well as some applications.

1. Review of Differentiable Manifolds and Differential Forms: a short review on the basics on manifolds, vector fields, and differential forms needed for this course.


3. Proper Actions: Bochner Linearization theorem, slices, tube theorem, orbit spaces, orbit types and stratification, principal and regular orbit types, blowups and desingularization.


2) J.J. Duistermaat & J.A.C. Kolk, Lie groups, Universitext, Springer-Verlag.
Instructor: Charles Rezk

Course description:
This is second semester course in algebraic topology. In the first semester (Math 525), invariants called homology groups were constructed in terms of the singular chain complex of a space. One of the themes of this course is thinking about the singular chain complex itself as a kind of invariant, from which other invariants (homology and cohomology groups, possibly with coefficients) can be derived, as well as additional structure on them (cup products, cohomology operations).

Course requirements:
Homework: There will be approximately six homework assignments, to be given out approximately once every two weeks.

Prerequisites: Math 525, or instructor consent.

Texts: The primary text will be:


This will be supplemented with additional course notes. Other references are:

- Bredon, *Geometry and Topology*.
- Bott & Tu, *Differential Forms in Algebraic Topology*.
- May, *A Consise Course in Algebraic Topology*.

Course topics:

- Singular cohomology.
- The cup product and the Künneth theorem.
- Čech cohomology and its relationship to singular cohomology.
- Poincaré Duality.
- Cohomology operations.
- Spectral sequences and applications.
FALL 2016

MATH 531

Analytic Theory of Numbers I

Section E1, CRN 30823

12-12:50 pm MWF, 341 Altgeld Hall

Kevin Ford

Math 540. Fall 2016.
Real Analysis.

Prof. Richard Laugesen  <Laugesen@illinois.edu>

Real analysis is the study of functions, especially their integrability and differentiability properties.

Classical real analysis, as taught at the undergraduate level in terms of Riemann integration and continuously differentiable functions, is completely inadequate for the modern needs of differential equations, functional analysis, probability theory, and so on.

This course develops modern integration theory (in Euclidean spaces and abstract measure spaces), and modern differentiation theory for functions of bounded variation. Then we develop $L^p$ theory, which provides a one-parameter family of norms for measuring the “size” of functions.

**Prerequisites:** Math 447 is the official prerequisite. Unofficially, students need a certain amount of mathematical maturity. If you have not studied metric spaces, then you should take Math 535 before attempting Math 540.

**Assessment:** Two midterms and a final exam. Weekly homework.

**Course website**  http://www.math.illinois.edu/~laugesen/

**Textbook**  
Download it free online  
[http://homepages.uconn.edu/~rib02005/real.html](http://homepages.uconn.edu/~rib02005/real.html)

**Supplementary reading (not required)**  
Complex Variables II

Instructor: Prof. Aimo Hinkkanen
Office: 345 Illini Hall, phone 244-7306, e-mail aimo@math.uiuc.edu
Lectures: Tuesdays and Thursdays 12.30–1.50 pm

Prerequisite: Math 542 or equivalent

The purpose of this course is to give the participants a broad view of many areas that are subjects of active research in contemporary complex analysis of one variable, giving the students the basis to continue their studies deeper in any direction.

Topics to be covered include, to the extent that time permits:
- conformal structures, Riemann surfaces, covering surfaces, the statement of the uniformization theorem, groups of Möbius transformations
- hyperbolic metric, the distance decreasing property of analytic functions
- subharmonic functions, Green’s functions, harmonic measure
- modules of path families, quasiconformal mappings: geometric, analytic, and metric definitions, basic properties, quasidisks and quasicircles, existence of solutions to the Beltrami equation, the parametric form of the measurable Riemann mapping theorem due to Ahlfors and Bers
- Nevanlinna’s value distribution theory, the Poisson–Jensen formula, the first and second fundamental theorems, applications
- basics of complex dynamics, Fatou and Julia sets, classification of dynamics, the Mandelbrot set

There is no single textbook for this course. The instructor will distribute his own lecture notes. The following books cover some of the areas (each book covers more material in its area than will be covered in this course).

Instructor: Zhong-Jin Ruan
Classroom: Altgeld Hall MWF 3:00-3:50 pm
Office: 353 Altgeld Hall
Email: ruan@math.uiuc.edu

This is an introductory course in Operator Theory and Operator Algebras.
In this course, we plan to cover

1) Geometric properties of Hilbert spaces
2) Compact and bounded linear operators on Hilbert spaces
3) Basic theory of Banach algebras and C*-algebras
4) Spectral theory of bounded normal operators
5) Unbounded operators on Hilbert spaces.

Prerequisite: Math 541.


References:

2) Fundamentals of the Theory of Operator Algebras I.
   Kadison and Ringrose
Course Description: An introduction to the study of dynamical systems. Considers continuous and discrete dynamical systems: linear and nonlinear differential equations, flows and maps on Euclidean space and other manifolds. Among other things, we will study the existence and uniqueness of solutions, dependence on initial conditions and parameters, linearization, stable and center manifold theorems. Discrete dynamics includes Bernoulli shifts, elementary Anosov diffeomorphisms and surfaces of sections of flows. Bifurcation phenomena in both continuous and discrete dynamics will be studied.

Texts: No text required: the course will be based on material from many books. A good reference book is the following:
Series: Texts in Applied Mathematics
Title: Ordinary Differential Equations with Applications
Author: Carmen Charles Chicone
Publisher: Springer

Prerequisite: Math 489 or consent from the instructor.
The course covers and develops techniques from functional analysis along with their implementation in the theory of partial differential equations. Time permitting we will cover the following topics.

**Coverage:** Short introduction to Banach and Hilbert space theory. Banach Fixed Point Theorem and applications to differential and integral equations.

The theory of $L^p$ spaces. Completeness, Duality, Reflexivity, Convolutions and Mollification, along with the basic inequalities that we use in PDEs.


Distributions and Fourier Transform on $\mathbb{R}^n$. $L^2$-based Sobolev spaces, Fundamental Solutions, Green’s Functions. Heat, Schrödinger or Linear Wave equation on $\mathbb{R}^n$ by inverting the Fourier transform.


Introduction to nonlinear partial differential equations.

**Prerequisites:** Math 447, Math 489, or consent of instructor. Math 540 would be useful, but is not required.

The course will be based on my lecture notes. The notes will be detailed and self contained. Recommended texts:
1. L. E. Evans, Partial Differential Equations.

For students which are not familiar with modern analysis techniques it may be helpful to consult from time to time the following books:
1. G. B. Folland, Real Analysis.
2. P. D. Lax, Functional Analysis.

The grade will be based on regular homework, participation and attendance.
FALL 2016

MATH 562

THEORY OF PROBABILITY II

Section F1, CRN 33567

11-12:20 PM TR, 170 Wohlers Hall

Professor Dey

Course Topics: This is the second half of the basic graduate course in probability theory. The goal of this course is to understand the basic theory of stochastic calculus. We will cover the following topics: (1) Brownian motion; (2) continuous time martingales; (3) Markov processes; (4) stochastic integrals; (5) Ito's formula; (6) representation of martingales; (7) Girsanov theorem and (8) stochastic differential equations. If time allows, we will give a brief introduction to mathematical finance.

Prerequisite:
1. Math 540 Real Analysis I - we will review measure theory topics as needed.
2. Math 541 is also nice to have, but not necessary.
3. Math 561 Probability Theory I - you should be willing to spend time and effort on this background material if necessary.

Grading: 50% of your grade will depend on homework assignment, and 50% will depend on a final paper, 3- to 6-page paper summarizing and analyzing a research article that is both interesting to you and related to probability.

Course Topics: This course is designed for both math and non-math graduate students. Measure theory is NOT a prerequisite for this course. However, you do need a basic knowledge of probability theory (math 461 or its equivalent).

The goal of this course is to reach fairly rigorous understanding of Markov chains and Markov processes. We are going to cover most of the materials from Norris' book, augmenting the text when necessary. Below is a rough list of some of the topics we will cover:

Strong Markov properties, recurrence and transience, invariant distributions, convergence and ergodicity, time reversal, Q-matrices, holding time, forward and backward equations, martingales and potential theory, queuing networks, Markov decision processes, Markov Chain and Monte Carlo techniques. Depending on the interest of the audience, we may cover some additional materials.

Text: J.R.Norris, Markov Chains, Cambridge University Press

Grading Policy: 70% of your grade will depend on homework assignment, and 30% will depend on a take home final exam.
Model theory studies mathematical structures that fit well into the framework of first order logic. These include algebraic structures such as groups, rings, fields, modules, as well as combinatorial/set theoretic ones, such as graphs and partial orders. The model-theoretic tools often reveal actually deep and important information about these structures, making model theory a highly applicable subject to other areas of mathematics, especially to algebra, algebraic geometry, algebraic number theory, and combinatorics.

One of the main objects model theory investigates in a given structure is the collection of definable sets and functions. Denoting by $M$ the underlying set of a structure $M$, a set $A \subseteq M^k$ is called definable in $M$ if the property that carves out $A$ from $M^k$ is expressible by the means of first order logic. A function $f : M^k \to M$ is called definable if its graph is definable (as a subset of $M^{k+1}$). We will develop tools for analyzing the definable sets and functions, including quantifier elimination, which, when true, allows removing quantifiers from the definitions of the definable sets and functions, limiting what they can be and making them much simpler to analyze. For instance, quantifier elimination for the field of real numbers makes the properties of semialgebraic sets rather transparent, yielding a natural and quick solution of Hilbert’s 17th Problem$^1$.

An example of a basic tool in model theory is passing to an elementary extension of a structure (e.g., its ultrapower), which can be viewed as a completion of the structure, in the same spirit as obtaining the reals from the rationals, but it preserves the combinatorial structure of definable sets and functions.

The course will culminate in the proof of Morley’s famous theorem, which is about categoricity of a theory $T$—the phenomenon of any two models of $T$ of the same (large enough) cardinality being isomorphic. For example, vector spaces and algebraically closed fields are categorical and both of these instances hinge on the existence of a basis with respect to a notion of dependence (linear for vector spaces, algebraic for fields). The proof of Morley’s theorem reveals that whenever the phenomenon of categoricity happens, it is because of the existence of a basis with respect to a certain notion of dependence, which is quite remarkable given that the structures under consideration are a priori rather arbitrary.

**Prerequisites:** Familiarity with basic syntax and semantics of first order logic (the first week of Math 570) will be assumed. Students who haven’t taken any course in logic can easily acquire this by reading the first pages of any text in logic, for example, Lou van den Dries’s or Anush Tserunyan’s lecture notes available in their respective websites.

**Required work:** Students will be required to write solutions to problems assigned during the course and may also be asked to present them on the board.

**Texts and references:** There is no required text; Ward Henson’s lecture notes will be made available by the instructor. Recent books that can be used as references include *Model Theory: An Introduction* by David Marker and *A Course in Model Theory* by Bruno Poizat.

---

$^1$A rational function over $\mathbb{R}$ is positive semi-definite iff it is a sum of squares of rational functions.
FALL 2016

MATH 580: Combinatorial Mathematics

Section D1, CRN 33562

11:00-11:50am MWF, 347 ALTGELD Hall

Jozsef Balogh
Web page: http://www.math.uiuc.edu/~jobal

SYLLABUS: This is a rigorous, graduate level introduction to combinatorics. It does not assume prior study, but requires mathematical maturity; it moves at a fast pace. The first third of the course is on enumeration. The second third covers graph theory. The remainder of the course considers some topics that are treated more in depth in advanced graduate courses (Math 581, 582, 583, 584): Ramsey theory, partially ordered sets, the probabilistic method and combinatorial designs (as time permits).

REQUIREMENTS: A raw score of 80% or higher guarantees an A while a score of 60% or higher guarantees a B- (grade drops by 5%). Additionally, for an A, in the final exam minimum 50%, for B+ (passing com) 40% required. (Near) weekly assignments. Each assignment will have 6 problems of your choice of 5/6 are graded. There are 12 homework assignments, each worth 6%, a midterm 8% and a final exam for 20%.
The grading: 80%−: A, 75%−: A−, 65%−: B+, 60%−: B, etc. Note that the writings of the solutions must have a high quality and typed; if the argument is messy or not typed then even if the solution is correct it could be returned without grading with 0 points.
Note that requirements are not as strict if somebody register as CS 571, which is for non-math major students recommended. Late homework policy: In case the homework is not submitted on time, it could be submitted for the next class, with losing 10% of the score. If there is official or medical reason, then try to notify me in advance via e-mail.

TEXTBOOK: The FALL 2015 edition of the text COMBINATORIAL MATHEMATICS (by Douglas West) will available at TIS Bookstore.

RESOURCES: Electronic mail is a medium for announcements and questions.

PREREQUISITES: There are no official prerequisites, but students need the mathematical maturity and background for graduate-level mathematics.
Section F1: 1pm MWF
Instructor: A. Kostochka, 234 Illini Hall, 265-8037, kostochk@math.uiuc.edu.

TOPICS: This is a companion course to Math 581 — Extremal Graph Theory. The two courses are independent. Structure of Graphs includes topics drawn from the following (not all will be covered).

Elementary Structural Concepts — structural and enumerative topics involving trees and related graphs, degree sequences, embeddings of graphs in product graphs, and the reconstruction problem (is $G$ reconstructible from the deck of subgraphs obtained by deleting a single vertex? . . . a single edge?). Graph packings and equitable colorings.

Connection and Cycles — min-max relations for connectivity and branchings, structure of k-connected graphs, Hamiltonian cycles and circumference, communication problems (gossip problem, etc.).

Topological Graph Theory — embeddings on surfaces (without edge crossings), characterizations and properties of graphs embeddable in the plane (separator theorems, Schnyder labelings), measures of non-planarity, voltage graphs and chromatic number of surfaces.

Matchings and joins — the structure of graphs with perfect matchings, the structure of solutions of the Chinese Postman Problem, the language of conservative weightings for finding maximal joins and minimum $T$-joins, cycle covers and nowhere-zero flows.

Graph Minors — treewidth and the minor order, some discussion of Robertson-Seymour Theorem (every minor-closed family of graphs has infinitely many minimal forbidden minors), forbidden and forced minors.

COURSE REQUIREMENTS: There will be 5 problem sets, each requiring 5 out of 6 problems for 50 points total; no exam. The problems require proofs related to or applying results from class.

PREREQUISITES: Familiarity with elementary graph theory. Math 580 provides sufficient preparation, as do most versions of Math 412. Interested students with no graph theory background may browse a basic text in advance, such as Diestel, Graph Theory, or the Math 412 text: West, Introduction to Graph Theory (Prentice Hall, 2001, first seven chapters). Important results needed from elementary graph theory will be reviewed.

TEXT: D. B. West, The Art of Combinatorics, Volume II: Structure of Graphs. For some topics, instructor’s supplements will be provided.